CALCULUS 03: The PRODUCT RULE

In a previous guide Calculus02, you learned about basic derivatives or differentiation.

Sometimes you need to find the derivative of something more complex, the PRODUCT, or multiple, of two functions.

An example is \( f(X) = (5+X^4)*(6X^2 + e^{3X}) \)

The easiest way (fewer errors) is NOT to expand it and differentiate it in parts, but to use THE PRODUCT RULE.

THE PRODUCT RULE.

If a function \( f(X) \) is equal to the product of two other functions of \( X \), say \( f(X) = g(X) \ast h(X) \),

Then the derivative is

\[
 f'(x) = g(X) \ast h'(X) \ast + \ast h(X) \ast g'(X)
\]

WORKED EXAMPLE

In our example \( g(X) = (5+X^4) \) and \( h(X) = (6X^2 + e^{3X}) \)

Hence

\[ g'(x) = 0 + 4\ast X^3 , \text{ simplifying to } g'(x) = 4\ast X^3 \]

and

\[ h'(X) = 6\ast 2\ast X^{(2-1)} + 3\ast e^{3X} \text{ simplifying to } h'(X) = 12\ast X^1 + 3\ast e^{3X} \]

Then substituting all these into \( f'(x) = g(X) \ast h'(X) + h(X) \ast g'(X) \)

We get \( f'(x) = \{(5+X^4)\ast (12\ast X^{(1)} + 3\ast e^{3X})\} + \{(6X^2 + e^{3X})\ast 4\ast X^3\}. \)

This DOES look complicated BUT can we expand this to simplify? Yes. Firstly, use the DISTRIBUTIVE LAW.

\[ f'(x) = \{60\ast X + 15\ast e^{3X} + 12\ast X^5 + 3\ast X^4 \ast e^{3X}\} + \{(6X^2 + e^{3X})\ast 4\ast X^3\} \]

Getting all the LIKE TERMS together. Chosen were the \( X^5 \) terms, then the \( e^{3X} \) terms. You could choose others.

\[ f'(x) = 36\ast X^5 + e^{3X} \ast(15 + 4\ast X^3 + 3\ast X^4 ) + 60\ast X \]

Don’t go any further, there are no more LIKE TERMS. That is enough! (*Check over the page)

Because functions which are the product of two other functions are used in many disciplines, but particularly Economics and Finance, the following video is recommended:

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(*) CHECK OF THE EXAMPLE PROBLEM ON PREVIOUS PAGE

You can check this on the “wolframalpha’ computational engine on http://www.wolframalpha.com/ as per the snip below. Phew, it worked.

The request was input as “differentiate f(X) = (5+X^4)*(6*X^2+e^(3X))”

Note that many systems use the cap symbol ^ (upper case 6 key) to indicate a power.

\[
\text{differentiate } f(X) = (5+X^4)(6X^2+e^{3X})
\]

\[
\begin{align*}
\text{Input interpretation:} \\
\text{differentiate } f(X) &= (5+X^4)(6X^2+e^{3X}) \\
\text{with respect to } &X \\
\text{Result:} \\
f'(X) &= 12X(3X^4 + 5) + e^{3X}(3X^4 + 4X^3 + 15) \\
\text{Alternate forms:} \\
36X^5 + e^{3X}(3X^4 + 4X^3 + 15) + 60X &= f'(X) \\
f'(X) &= 12X(3X^4 + 5) + e^{3X}(3X^4 + 4X^3 + 15) \\
f'(X) &= 36X^5 + 3e^{3X}X^4 + 4e^{3X}X^3 + 60X + 15e^{3X}
\end{align*}
\]