CALCULUS 04: The QUOTIENT RULE

In a previous guide Calculus02, you learned about basic derivatives or differentiation.

Sometimes you need to find the derivative of something more complex, the QUOTIENT, or division, of two functions.

An example is \( f(X) = \frac{5+X^4}{6X^2 + e^{3X}} \)

The easiest way (fewer errors) is NOT to expand it and differentiate it in parts, but to use THE QUOTIENT RULE.

**THE QUOTIENT RULE.**

If a function \( f(X) \) is equal to the quotient of two other functions of \( X \), say \( f(X) = \frac{g(X)}{h(X)} \)

Where the top function, the numerator is \( g(X) \)

The bottom function, the denominator, is \( h(X) \)

Then the derivative \( f'(x) = \frac{\{g(X) \cdot h'(X)\} - \{h(X) \cdot g'(X)\}}{h(X)^2} \)

Note the pattern:

{top function * derivative of bottom function} MINUS {bottom function * derivative of top function}

All divided by the square of the bottom function.

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For reasons that will become clear, the examples given to test you at university are usually kept very simple if you are not permitted to use programmable calculators or to use computers.

To prove this, let’s have a not-so-simple example \( f(X) = \frac{g(X)}{h(X)} \), with \( g(X) = (5+X^4) \) and \( h(X) = (6X^2 + e^{3X}) \)

Hence \( g'(x) = 0 + 4 \cdot X^3 \), simplifying to \( g'(x) = 4 \cdot X^3 \)

and \( h'(X) = 6 \cdot 2 \cdot X^{(2-1)} + 3 \cdot e^{3X} \) simplifying to \( h'(X) = 12 \cdot X^{1} + 3 \cdot e^{3X} = 12 \cdot X + 3 \cdot e^{3X} \)

and \( h(X)^2 = (6X^2 + e^{3X})^2 \) Let’s not simplify this at this stage. It may not be necessary.

Then substituting all these into \( f'(x) = \frac{(5+X^4) \cdot (12 \cdot X + 3 \cdot e^{3X}) - (6X^2 + e^{3X}) \cdot (4 \cdot X^3)}{(6X^2 + e^{3X})^2} \)

\( Eek! \)

This DOES look complicated BUT can we expand at least the top part to simplify it a bit?

Yes. Firstly, use the DISTRIBUTIVE LAW.

Getting all the LIKE TERMS together. Chosen were \( e^{3X} \) terms then the rest had a common factor of 12*X. You could choose others.

The top part becomes \( 12 \cdot X \cdot (X^5 - 5) + e^{3X} \cdot (3 \cdot X^4 + 4 \cdot X^2 -15) \)

So \( f'(x) = \frac{12 \cdot X \cdot (X^5 - 5) + e^{3X} \cdot (3 \cdot X^4 + 4 \cdot X^2 -15)}{(6X^2 + e^{3X})^2} \)

Don’t go any further. That is enough! (*Check over the page)

Because functions which are the quotient of two other functions are not very common, the following video with some BASIC examples is recommended:

http://patrickjmt.com/derivatives-the-quotient-rule-a-few-basic-examples/
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(∗) Check the problem.

You can check this on the “wolframalpha’ computational engine on http://www.wolframalpha.com/ as per the snip below. Phew, it worked.

The request was input as “differentiate f(X) = (5 + X^4) / (6 * X^2 + e^(3 * X))”

The placing of brackets is important
Note that many systems use the cap symbol ^ (upper case 6 key) to indicate a power.

\[
\text{differentiate } f(X) = \frac{5 + X^4}{6 X^2 + e^{3X}}
\]

\[
f'(X) = \frac{12 X (X^4 - 5) + e^{3X} (-3 X^4 + 4 X^3 - 15)}{(6 X^2 + e^{3X})^2}
\]

Alternate forms:
\[
f'(X) = \frac{12 X (X^4 - 5) + e^{3X} (-3 X^4 + 4 X^3 - 15)}{(6 X^2 + e^{3X})^2}
\]
\[
f'(X) = \frac{12 X^5 - 3 e^{3X} X^4 + 4 e^{3X} X^3 - 60 X - 15 e^{3X}}{(6 X^2 + e^{3X})^2}
\]

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